

Aspects of $(0,2)$ theories

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A progress report on $d=2$ QFT with $(0,2)$ supersymmetry

- ★ Gross, Harvey, Martinec & Rohm, *Heterotic string theory (I)*
NPB**256**, received Feb. 26 1985

There are many paths to (0,2) theories

- ★ critical heterotic strings with $\mathbb{R}^{1,3}$ N=1 superPoincaré
- ★ heterotic stringy geometry for $\mathcal{E} \rightarrow X$
- ★ surface defects/strings in d=4 N=1 gauge theory
- ★ AdS₃/CFT₂ and (2,0) compactification
- ★ last refuge of holomorphy in SUSY QFT

Plan

- ① general properties
- ② heterotic geometries
- ③ linear sigma models
- ④ accidents
- ⑤ outlook

(0,2) supercurrents live in an \mathcal{S} -multiplet

[Dumitrescu&Seiberg,1106.0031]

- ★ $\mathbb{R}^{1,1}$: $x^\mu = (x^0, x^1)$ or light-cone $x^{\pm\pm}$
- ★ S_+^μ, \bar{S}_+^μ conserved supercurrents $\rightarrow Q_+, \bar{Q}_+$
- ★ energy-momentum tensor $T^{\mu\nu} \rightarrow P^\mu$

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- ★ weak assumptions $\implies T, S, \bar{S} \subset \mathcal{S}$ -supermultiplet

$$\begin{aligned}\{Q_+, \bar{S}_{+\pm\pm}\} &= -T_{\pm\pm++} \pm i\partial_{\pm\pm} j_{++} & \{Q_+, S_{++++}\} &= 0 \\ \{Q_+, S_{+--}\} &= i\bar{C} .\end{aligned}$$

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- ★ spin 1 operator j_{++} not necessarily conserved
- ★ $\bar{C} \neq 0 \implies$ deformed (0,2) supercurrent algebra, a UV property

Many (0,2) QFTs have \mathcal{R} -multiplet

[Dumitrescu&Seiberg,1106.0031]

★ $\bar{C} = 0$

★ conserved R-current $j^\mu \leftrightarrow j_{\pm\pm} \rightarrow$ conserved charge R

★ $j^\mu, S_+^\mu, \bar{S}_+^\mu, T \in \mathcal{R}$ -multiplet \implies standard (0,2) SUSY algebra

$$\begin{aligned} \{Q_+, \bar{Q}_+\} &= -4P_{++} & [R, Q_+] &= -Q_+ & [R, \bar{Q}_+] &= +\bar{Q}_+ \\ \{Q_+, Q_+\} &= 0 & \{\bar{Q}_+, \bar{Q}_+\} &= 0 \end{aligned}$$

★ a key property: $\bar{Q}_+^2 = 0$

★ \bar{C} can be generated by quantum corrections, e.g. (0,2) \mathbb{P}^1 NLSM

(0,2) SCFTs are the building blocks for (0,2) QFTs

- ★ $T_{\mu}^{\mu} = 0$ on Euclidean world-sheet

$$\text{Vir}_{\mathbf{c}} \oplus \overline{\text{N=2 sVir}_{\mathbf{\bar{c}}}} \quad T(z) \quad ; \quad \bar{J}(\bar{z}), G^{\pm}(\bar{z}), \bar{T}(\bar{z})$$
$$h \quad \quad \quad \bar{q} \quad \quad \quad \bar{h}$$

- ★ we will discuss unitary and compact SCFTs
- ★ characterization
 - symmetries, e.g. left-moving Kac-Moody algebra $\supset U(1)_L$ current J
 - elliptic genus $Z = \text{Tr}_{RR}(-1)^F q^H \bar{q}^{\bar{H}} y^{J_0}$
 - spectrum of marginal operators [depends on moduli!]
 - 2 and 3-point functions [super-primary not sufficient!]

Marginal (0,2) deformations are characterized

- ★ SUSY marginal deformations via conformal perturbation theory

[Bertolini et al,1405.4266], cf N=1 d=4 [Green et al,1005.3546]

$$\Delta S = \lambda \int d^2z \{Q_+, U\} + \text{h.c.}$$

U a chiral primary operator $h = 1, \bar{h} = \bar{q}/2 = 1/2$

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- ★ unitarity [$\bar{h} \geq \bar{q}/2$] \implies λ at worst marginally irrelevant
- ★ obstructions:
 - “D-term” $(\mathcal{J}_{\text{KM}}(z), U, \bar{U}) \rightarrow$ long multiplet \implies λ breaks KM
 - “F-term” $(U_{\bar{q}=1}, F_{\bar{q}=2}) \rightarrow$ long multiplet
- ★ some consequences:
 - Kähler moduli space
 - no F-term obstructions for $\bar{c} < 6$

RG flows satisfy some simple constraints

- ★ relevant $\Delta S = \lambda \int d^2z \{Q_+, U\} + \text{h.c.}$; U c.p. with $\bar{q} < 1$
- ★ $(c - \bar{c})$ is RG-invariant; $\dot{c} < 0$
- ★ $\bar{c}_{\mathbb{R}}$ via **c-extremization** [Benini&Bobev,1211.4030,1302.4251]
- ★ elliptic genus is RG-invariant
- ★ holomorphic 1/2-twisted chiral ring of \bar{Q}_+ -closed operators
[Witten,0504078],[Tan&Yagi,08014742],[Borisov&Kaufmann,1102.5444]
- ★ **topological heterotic rings** generalize A and B model of (2,2)
[Adams,Basu&Sethi,0309226],[Adams,Distler&Ernebjerg,0506263]
- ★ but watch out for accidents!

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(0,2) NLSMs yield a geometric realization

★ geometry [Hull&Witten,PLB'85] :

- X complex, Hermitian ω , $H = i(\bar{\partial} - \partial)\omega$
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★ conformal invariance for $\dim_{\mathbb{C}} X = 3 \implies$

[Strominger;Hull;Sen,'86],[Groot Nibbelink&Horstmeyer,1203.6827]

- K_X holomorphically trivial and \mathcal{E} stable
- (X, ω, Ω) define $SU(3)$ structure + conformal balance
- $dH = \frac{\alpha'}{4} [\text{tr } \mathcal{R}^2 - \text{tr } \mathcal{F}^2]$

The standard embedding is a heterotic geometry

- ★ (X, ω, Ω) is CY 3-fold; $\mathcal{E} = T_X$; $H = 0 \implies$
 - (2,2) SCFT $c = \bar{c} = 9$ $\bar{q} \in \mathbb{Z}$
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- ★ W encodes obstructions to $H^1(\text{End } T_X)$
 - classical via alg-geom
 - quantum via world-sheet instantons

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- ★ world-sheet instantons can ruin conformal invariance
zoo or bestiary?

Torsional geometries are more mysterious

- ★ X topologically non-Kähler e.g. odd b_1
- ★ string-scale cycles $\implies \alpha'$ expansion formal
- ★ known examples: $X = T^2$ principal bundle over K3

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[Dasgupta et al,9908088],[Fu&Yau,0604063],[Becker et al,0604137]
- ★ torsional geometries with N=2 spacetime supersymmetry
[IVM,Minasian&Theisen,1206.1417]
 - classified
 - exhibit (0,2+4) worldsheet susy — cf [Banks&Dixon,NPB'88]
 - conjecture: IIA duals are K3-fibered but not elliptically fibered

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- ③ linear sigma models
- ④ accidents
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Linear sigma models offer key insights

- ★ two-dimensional gauge theories with (0,2) susy [\[Witten,9301042\]](#)
- ★ simple UV physics
 - chiral bosonic multiplets Φ and fermi multiplets $\Gamma^A = \gamma_-^A + \dots$
 - gauge group G and holomorphic potentials $E^A(\Phi)$, $J_A(\Phi)$, $J \cdot E = 0$
 - “phases” determined by Fayet-Iliopoulos parameters
 - geometric phases include $X \subset V_{\text{toric}}$ & $\mathcal{E} \rightarrow X$ **monad bundle**

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- ★ rich IR physics
 - vast classes of (2,2) and (0,2) SCFTs
 - interpolate between disparate descriptions — CY/LG correspondence
- ★ the art of the linear sigma model : lift IR queries to UV fields

Developments in $(0,2)$ linear sigma model technology

★ elliptic genus [[Gadde&Gukov,1305.0266](#)],[[Benini et al,1308.4896](#)]

⇒ probe novel $(0,2)$ SCFTs, e.g. [[Gadde et al,1404.5314](#)],[[Haghighat et al,1412.3152](#)]

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 - \implies (X, \mathcal{E}) geometries safe from world-sheet instantons

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- ★ $\{\text{holomorphic parameters}\}/\{\text{redefinitions}\}$ for deformations of $(2,2)$
[Kreuzer et al,1001.2104]
⇒ $(0,2)$ mirror map for pairs $(X, \mathcal{E}), (X^\circ, \mathcal{E}^\circ)$ [IVM&Plesser,1003.1303]

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⇒ quantum sheaf cohomology for toric varieties [Guffin et al,1110.3751]
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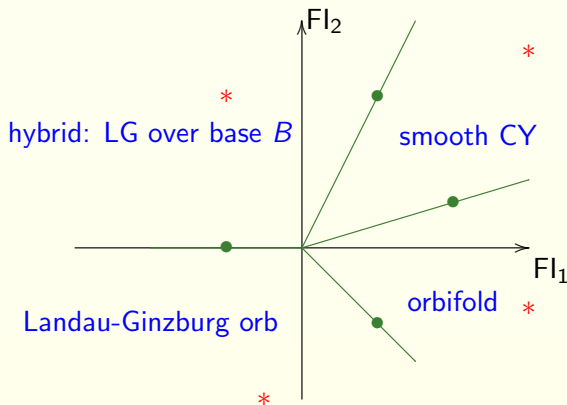
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- ★ torsional linear sigma models[Adams et al,0611084,1206.5815],[Quigley et al,1206.3228,1212.1212]

(0,2) marginal deformations via (2,2) linear sigma model

- ★ marginal deformations depend on the phase

[Kachru&Witten,9307038],[Distler&Kachru,9309110],[Aspinwall et al,1008.2156],[Bertolini et al,1307.7064]



- ★ a rich laboratory for explorations, e.g. (0,2) McKay [Aspinwall&Gaines,1404.7802]

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Life without accidents would be nice

- ★ (0,2) Landau-Ginzburg theory with $U(1)_L \times U(1)_R$ symmetry

$$L = \text{free kinetic term} + \int d\theta \sum_A \Gamma^A J_A(\Phi; \alpha) + \text{h.c.}$$

- ★ quasi-homogeneity: $J_A(t^q \Phi; \alpha) = t^{-Q_A} J_A(\Phi; \alpha)$
- ★ charges (q, Q) determine family of LG theories
- ★ generic $J_A \implies U(1)_L \times U(1)_R$ symmetry
- ★ IR SCFT with (c, \bar{c}) & elliptic genus determined by (q, Q)

There are accidents in $(0,2)$ RG flows

[Bertolini et al,1405.4266]

- ★ LG kinetic term irrelevant and slaved to superpotential
- ★ $J_A(\Phi; \alpha)$ and $J_A(\Phi; \alpha')$ are IR-equivalent if linked by field redefinition
- ★ symmetries of $J_A(\Phi; \alpha)$ depend on α
 - \implies c -extremization can lead to $c(\alpha) \neq c(\alpha')$!
 - \implies UV parameter space stratified according to basin of attraction
 - \implies naive c may not have realization for any α
- ★ how can we ensure that accidents do not occur?

Outlook

- ★ impressive results for just two supercharges
- ★ many challenges
- ★ some key directions
 - when does a $(0,2)$ linear sigma model yield a heterotic CFT?
 - what is $(0,2)$ mirror symmetry beyond deformations of $(2,2)$?
 - when is a $(0,2)$ RG-flow accident-free?
 - which torsional geometries yield heterotic CFTs?
 - what sorts of geometric transitions occur in $(0,2)$?
 - are there nice local models?
 - is there a class of $(0,2)$ “minimal models”?
- ★ hope: new sources of $(0,2)$ questions may also yield new $(0,2)$ answers
- ★ a little $(0,2)$ book is in the works!