

# Heterotic flux vacua and their IIA duals

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a résumé of ArXiv:1206.1417, with R. Minasian and S. Theisen

# Motivation

Heterotic string offers a rich interplay of geometry and physics  
... and thus lots of questions!

- which geometries lead to perturbative string vacua (i.e. CFTs)?
- which of those exist non-perturbatively?
- what is the structure of moduli spaces and possible transitions?

# Motivation

$N=1, \mathbb{R}^{1,3}$  vacua have been a major focus

- phenomenology (heterotic and F)
- stringy geometry (worldsheet instantons, quantum sheaf cohomology)
- flux and non-geometric vacua

this talk :  $N=2 \mathbb{R}^{1,3}$  perturbative heterotic vacua and their duals

- quantum corrections under better control, yet far from trivial
- an elegant targetspace geometry
- more general perspective on heterotic / IIA duality

# Outline

- N=2 heterotic compactification: CFT and geometry
- Flux and its consequences
- A  $d = 8$  perspective
- IIA / heterotic duality
- Bright future

# Worldsheet vs spacetime supersymmetry

Classic result 1 [Banks, Dixon, Friedan & Martinec, '88]

critical perturbative heterotic  $\mathbb{R}^{1,3}$  vacuum has N=1 SUSY  
 $\iff$   
(0,1) internal SCFT  $\rightarrow$  (0,2) SCFT,  $\bar{q} \in \mathbb{Z}$

Classic result 2 [Hull&Witten, '85; Sen, '86; Strominger, '86; Hull&Townsend, '86]

if **geometric** realization by (0,1) NLSM with target  $X$  then:

$$\begin{array}{ll} X \text{ complex 3-fold,} & K_X = \mathcal{O}_X; \\ d(J \wedge J e^{-2\varphi}) = 0, & \mathcal{H} = i(\bar{\partial} - \partial)J; \\ E \rightarrow X, \text{ HYM } \mathcal{A}, & d\mathcal{H} = \frac{\alpha'}{4}(\text{tr}\{\mathcal{R}_+^2\} - \text{tr}\{\mathcal{F}^2\}); \\ \mathfrak{g}_E \subset \mathfrak{so}(32) \text{ or } \mathfrak{so}(16)^2, & w_1(E) = w_2(E) = 0. \end{array}$$

# Worldsheet vs spacetime supersymmetry

Classic result 3 [Banks&Dixon, '88; Lauer, Lüst & Theisen, '88]

perturbative heterotic  $\mathbb{R}^{1,3}$  vacuum has  $N=2$  SUSY

$\iff$

$$\{N = 2\}_{\bar{c}=9} \text{ SCA} \rightarrow \{N = 2\}_{\bar{c}=3} + \{N = 4\}_{\bar{c}=6}.$$

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**Result** : if **geometric** realization by NLSM with target  $X$  then:

$$\begin{aligned} X \xrightarrow{\pi} M \text{ is principal } T^2 \text{ bundle,} & \quad M \text{ is K3, CY } \widehat{g}; \\ g = \pi^* e^{2\varphi} \widehat{g} + \mathcal{G}_{IJ} \Theta^I \Theta^J, & \quad \Theta^I = d\theta^I + \pi^* A^I; \\ F^I = dA^I \text{ are ASD,} & \quad \mathcal{H} = \widehat{\mathcal{H}} - \mathcal{G}_{IJ} \Theta^I F^J; \\ \mathcal{A} = \pi^* \widehat{\mathcal{A}} + \mathbf{a}_I \Theta^I & \quad \widehat{\mathcal{A}} \text{ HYM on } M, \\ \mathbf{a}_I \text{ are the Wilson lines,} & \quad \mathcal{G} \text{ is constant } T^2 \text{ metric.} \end{aligned}$$

cf heterotic fluxes [Dasgupta, Rajesh, Sethi '99; Goldstein&Prokushkin '02, Becker et al '06; Fu&Yau '06]

## Flux and its consequences

- constraints on K3 geometry (cf Abelian instantons on K3 [Honecker '06] )  
K3  $M$ , CY  $\widehat{g}$  determined by  $J, \Omega$  in  $H^2(M)$   
 $X \rightarrow M$  principal  $T^2$  bundle,  $F^I = dA^I \in H^2(M, 2\pi\mathbb{Z})$   
 $F^I$  ASD  $\iff J \wedge F^I = \Omega \wedge F^I = 0 \implies$  restrictions on  $\widehat{g}$   
 $\delta\widehat{B} \sim \epsilon_I F^I \implies \delta\widehat{B} \propto F^I$  not a deformation  
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- constraints on  $T^2$

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flux quantization  $\implies b$  and  $\mathcal{G}_{IJ}^* \equiv \mathcal{G}_{IJ} - \frac{\alpha'}{4} \text{tr}\{\mathbf{a}_I \mathbf{a}_J\}$  quantized.

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We can count first order deformations;

N=2 spacetime SUSY ensures these are integrable

flux compactification  $\iff X$  is non-trivial  $T^2$  bundle

# The $d = 8$ perspective

A gradual compactification [\[Kachru&Vafa '95\]](#)

- 1 compactify heterotic string on  $T^2$   
 $\implies d = 8$  SUGRA + SYM with gauge group  $G$
- 2 compactify further on a K3  $M$  with  $k = 24$  instanton  $\subset G$   
 $\implies d = 4$  N=2 SUGRA + axio-dilaton vector +  $H \subset G$  SYM  
+  $N_H^0$ -neutral +  $N_H^+$ -charged hypermultiplets

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**This is a sign of a heterotic flux vacuum!**

# The $d = 8$ perspective

Classic example [\[Kachru&Vafa '95\]](#)

- 1 compactify on  $T^2$  with  $\tau = \rho$ ,  $G = U(1) \times SU(2) \times E_8 \times E_8$
- 2 instanton :  $SU(2)_{k=4} \times SU(2)_{k=10} \times SU(2)_{k=10} \subset SU(2) \times E_8 \times E_8$   
gauge group:  $H = U(1)^2 \times E_7^2 + \text{graviphoton} + \text{axio-dilaton vector}$
- 3 on Higgs branch:  $H = U(1)^2$ ,  $N_H^0 = 129$   
 $\leftrightarrow$  IIA dual :  $Y_{12} \subset \mathbb{P}_{11226}^4$  ; detailed tests [\[Kaplunovsky,Louis & Theisen '95\]](#)

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**Result:** this is a heterotic flux vacuum!

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**Result:** heterotic flux vacua lead to many more examples with small  $H$ ;  
e.g. potential duals of K3-fibered CYs with  $h^{1,1} = 2$ ,  $h^{1,2} = 44, 62$   
(codim 2 complete intersections [Klemm, Kreuzer, Riegler & Scheidegger '04] )

## IIA / heterotic flux duality

Classic result 4 [Aspinwall & Louis '95; Morrison & Vafa '96; Aspinwall & Gross '96]

perturbative heterotic compactification  $\leftrightarrow$  IIA on smooth CY 3-fold  $Y$   
weak coupling  $\leftrightarrow$  a large radius limit,

$\implies Y$  is K3-fibered; Kähler class dual to generic fiber maps to axio-dilaton; moreover, if heterotic theory has a  $d = 6$  decompactification limit, then  $Y$  has a compatible elliptic fibration with section.

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**Proposal :** Let  $Y$  be a K3-fibered CY 3-fold without compatible elliptic fibration with section; its weakly-coupled heterotic dual, if it exists, will be an N=2 heterotic flux vacuum.

# Outlook

- Unlike their  $N = 1$  relatives,  $N = 2$  perturbative heterotic flux compactifications are quite constrained and controlled.
- Duality offers hope of determining non-perturbative corrections.
- Many tractable examples for study and exploration!
- We should test proposed dual pairs by examining the vector moduli space.
- It would be nice to develop a better picture of heterotic T-duality orbits [Evslin & Minasian '09]; interesting effects on the IIA side?
- What is the F-theory perspective on this class of theories?