Linear Sigma Models and Heterotic Moduli Spaces

Ilarion V. Melnikov

Albert Einstein Institute
Max Planck Institute for Gravitational Physics
Golm, Germany

based on work with M. Kreuzer, J. McOrist, and M.R. Plesser

Motivation

A future textbook problem

Given a perturbative heterotic string background with $d = 4$, $N = 1$ super-Poincaré invariance, determine

1. moduli space & massless spectrum;
2. Yukawa coupling dependence on moduli fields;
3. the singular locus of CFT.

Extra credit

Apply your results to

- issues in moduli stabilization;
- non-perturbative effects in heterotic string theory;
- quantum geometry.

Motivation

At present, difficult even with "textbook" starting point:

- Calabi-Yau three-fold M, a hypersurface in a toric variety V;
- standard embedding, i.e. holomorphic bundle $\mathcal{E} = T_M$.

Motivation

At present, difficult even with "textbook" starting point:

- Calabi-Yau three-fold M, a hypersurface in a toric variety V;
- standard embedding, i.e. holomorphic bundle $\mathcal{E} = T_M$.

Difficulty: bundle deformations and quantum corrections

Motivation

At present, difficult even with "textbook" starting point:

- Calabi-Yau three-fold M, a hypersurface in a toric variety V;
- standard embedding, i.e. holomorphic bundle $\mathcal{E} = T_M$.

Difficulty: bundle deformations and quantum corrections

Some special cases:

- special points in moduli space admit exact (0,2) SCFT description;
- $\mathcal{E} = T_M \implies$ (2,2) world-sheet SUSY & mirror symmetry;
- If (2,2) theory admits gauged linear sigma model description, can study *the (0,2) GLSM subspace of deformations*.

Motivation

At present, difficult even with "textbook" starting point:

- Calabi-Yau three-fold M, a hypersurface in a toric variety V;
- standard embedding, i.e. holomorphic bundle $\mathcal{E} = T_M$.

Difficulty: bundle deformations and quantum corrections

Some special cases:

- special points in moduli space admit exact (0,2) SCFT description;
- $\mathcal{E} = T_M \implies$ (2,2) world-sheet SUSY & mirror symmetry;
- If (2,2) theory admits gauged linear sigma model description, can study *the (0,2) GLSM subspace of deformations*.

A guiding question:

how does mirror symmetry extend to (0,2) GLSM deformations?

Results from the (0,2) GLSM moduli space

Results from the (0,2) GLSM moduli space

1. (0,2) deformations that are *not* lifted by instantons.

   [Silverstein + Witten 1995, Berglund et al 1995, Basu + Sethi 2003, Beasley + Witten 2003]

Results from the (0,2) GLSM moduli space

1. (0,2) deformations that are *not* lifted by instantons.

   [Silverstein + Witten 1995, Berglund et al 1995, Basu + Sethi 2003, Beasley + Witten 2003]

2. Yukawa couplings
   - topological heterotic rings [Adams, Basu+Sethi 2003, Adams, Distler + Ernebjerg 2005]
   - via A/2 and B/2 half-twisted theories

     [Katz+Sharpe 2004, Guffin+ Katz 2007, McOrist + IVM, 2007, 2008]

Results from the (0,2) GLSM moduli space

1. (0,2) deformations that are *not* lifted by instantons.

   [Silverstein + Witten 1995, Berglund et al 1995, Basu + Sethi 2003, Beasley + Witten 2003]

2. Yukawa couplings
   - topological heterotic rings [Adams, Basu+Sethi 2003, Adams, Distler + Ernebjerg 2005]
   - via A/2 and B/2 half-twisted theories

     [Katz+Sharpe 2004, Guffin+ Katz 2007, McOrist + IVM, 2007, 2008]

3. singular loci in moduli space [McOrist + IVM 2008, IVM + Plesser 2010]
   - points where SCFT expected to be singular
   - interpolate between (2,2) singular loci and large radius bundle singularities

Results from the (0,2) GLSM moduli space

1. (0,2) deformations that are *not* lifted by instantons.

   [Silverstein + Witten 1995, Berglund et al 1995, Basu + Sethi 2003, Beasley + Witten 2003]

2. Yukawa couplings
   - topological heterotic rings [Adams, Basu+Sethi 2003, Adams, Distler + Ernebjerg 2005]
   - via A/2 and B/2 half-twisted theories

     [Katz+Sharpe 2004, Guffin+ Katz 2007, McOrist + IVM, 2007, 2008]

3. singular loci in moduli space [McOrist + IVM 2008, IVM + Plesser 2010]
   - points where SCFT expected to be singular
   - interpolate between (2,2) singular loci and large radius bundle singularities

4. conjecture for a (0,2) mirror map [IVM + Plesser 2010]
   - suggested by form of algebraic coordinates
   - $(M, \mathcal{E}) \leftrightarrow (M^\circ, \mathcal{E}^\circ)$
   - a check: map exchanges singular loci

The GLSM: a $d = 2$ (2,2) SUSY gauge theory [Witten 1993]

- gauge group $G = \mathrm{U}(1)^r \times$ finite abelian group
- charged chiral matter multiplets Z_0, Z_ρ, $\rho = 1, \ldots, n$
- charges Q_0^a, Q_ρ^a, with $Q_0^a = -\sum_\rho Q_\rho^a$

The GLSM: a $d = 2$ (2,2) SUSY gauge theory [Witten 1993]

- gauge group $G = \mathrm{U}(1)^r \times$ finite abelian group
- charged chiral matter multiplets Z_0, Z_ρ, $\rho = 1, \ldots, n$
- charges Q_0^a, Q_ρ^a, with $Q_0^a = -\sum_\rho Q_\rho^a$

holomorphic parameters:

- FI+θ-angle terms in twisted superpotential $T_a \equiv e^{-2\pi r^a + i\theta^a}$
- coefficients A_m in chiral superpotential $W(Z) = Z_0 F(Z)$,

$$F(Z) = \sum_{m=0}^{u} A_m \prod_\rho Z_\rho^{P_{m\rho}+1}, \quad P_{m\rho} = \begin{cases} 0 \text{ if } \; \mathrm{m{=}0}, \\ \Pi_{m\rho} \text{ otherwise.} \end{cases}$$

The GLSM: a $d = 2$ (2,2) SUSY gauge theory [Witten 1993]

- gauge group $G = \mathrm{U}(1)^r \times$ finite abelian group
- charged chiral matter multiplets Z_0, Z_ρ, $\rho = 1, \ldots, n$
- charges Q_0^a, Q_ρ^a, with $Q_0^a = -\sum_\rho Q_\rho^a$

holomorphic parameters:

- FI+θ-angle terms in twisted superpotential $T_a \equiv e^{-2\pi r^a + i\theta^a}$
- coefficients A_m in chiral superpotential $W(Z) = Z_0 F(Z)$,

$$F(Z) = \sum_{m=0}^{u} A_m \prod_\rho Z_\rho^{P_{m\rho}+1}, \quad P_{m\rho} = \begin{cases} 0 \text{ if } & \text{m=0}, \\ \Pi_{m\rho} & \text{otherwise.} \end{cases}$$

Q_ρ^a, $\Pi_{m\rho}$ determined by combinatorics of a *reflexive polytope*:

- $\Pi_{m\rho}$: rank $d = n - r$ integral $u \times n$ matrix with entries ≥ -1;
- $\{Q_\rho^1, \ldots, Q_\rho^r\}$: integral basis for kernel of $\Pi_{m\rho}$.

The (2,2) GLSM & geometry

Combinatorics of $\Pi_{m\rho}$, $Q^a_\rho \to$ geometry:

- Z_ρ are projective coordinates for d-dimensional compact toric variety $V = \{\mathbb{C}^n - \Delta\}/G_{\mathbb{C}}$;
- $M = \{F = 0\} \subset V$ is a Calabi-Yau hypersurface in V.

The (2,2) GLSM & geometry

Combinatorics of $\Pi_{m\rho}$, $Q^a_\rho \to$ geometry:

- Z_ρ are projective coordinates for d-dimensional compact toric variety $V = \{\mathbb{C}^n - \Delta\}/G_{\mathbb{C}}$;
- $M = \{F = 0\} \subset V$ is a Calabi-Yau hypersurface in V.

GLSM/geometry connection:

- can choose T_a so that at low energy GLSM reduces to NLSM with target-space M.
- just one of the *phases* of the GLSM.

The (2,2) GLSM & geometry

Combinatorics of $\Pi_{m\rho}$, $Q^a_\rho \to$ geometry:

- Z_ρ are projective coordinates for d-dimensional compact toric variety $V = \{\mathbb{C}^n - \Delta\}/G_{\mathbb{C}}$;
- $M = \{F = 0\} \subset V$ is a Calabi-Yau hypersurface in V.

GLSM/geometry connection:

- can choose T_a so that at low energy GLSM reduces to NLSM with target-space M.
- just one of the *phases* of the GLSM.

The GLSM parameters

- T_a : Kähler deformations of $V \longrightarrow$ Kähler deformations of M
- A_m : complex structure deformations of M

The (2,2) GLSM moduli space

The (2,2) GLSM moduli space

- T_a : toric Kähler deformations $\qquad H^{1,1}_{\text{toric}}(M) \subseteq H^{1,1}(M)$

The (2,2) GLSM moduli space

- T_a : toric Kähler deformations $\qquad H^{1,1}_{\text{toric}}(M) \subseteq H^{1,1}(M)$

  Toric divisors need not intersect $M \implies$ some T_a are redundant.

The (2,2) GLSM moduli space

- T_a : toric Kähler deformations $\quad H^{1,1}_{\text{toric}}(M) \subseteq H^{1,1}(M)$

  Toric divisors need not intersect $M \implies$ some T_a are redundant.
- A_m : polynomial c-x structure defs $\quad H^{d-2,1}_{\text{poly}}(M) \subseteq H^{d-2,1}(M)$

The (2,2) GLSM moduli space

- T_a : toric Kähler deformations $\qquad H^{1,1}_{\text{toric}}(M) \subseteq H^{1,1}(M)$

  Toric divisors need not intersect $M \implies$ some T_a are redundant.
- A_m : polynomial c-x structure defs $\qquad H^{d-2,1}_{\text{poly}}(M) \subseteq H^{d-2,1}(M)$

  Redefinitions $Z_\rho \mapsto U_\rho(Z)$ act on $A_m \implies$ some A_m are redundant.

The (2,2) GLSM moduli space

- T_a : toric Kähler deformations $H^{1,1}_{\text{toric}}(M) \subseteq H^{1,1}(M)$

  Toric divisors need not intersect $M \implies$ some T_a are redundant.

- A_m : polynomial c-x structure defs $H^{d-2,1}_{\text{poly}}(M) \subseteq H^{d-2,1}(M)$

  Redefinitions $Z_\rho \mapsto U_\rho(Z)$ act on $A_m \implies$ some A_m are redundant.

  Example: $Z_\rho \mapsto u_\rho Z_\rho \implies A_m \mapsto A_m \times \prod_\rho u_\rho^{P_{m\rho}+1}$

The (2,2) GLSM moduli space

- T_a : toric Kähler deformations $\qquad H^{1,1}_{\text{toric}}(M) \subseteq H^{1,1}(M)$

  Toric divisors need not intersect $M \implies$ some T_a are redundant.

- A_m : polynomial c-x structure defs $\qquad H^{d-2,1}_{\text{poly}}(M) \subseteq H^{d-2,1}(M)$

  Redefinitions $Z_\rho \mapsto U_\rho(Z)$ act on $A_m \implies$ some A_m are redundant.

  Example: $\quad Z_\rho \mapsto u_\rho Z_\rho \implies A_m \mapsto A_m \times \prod_\rho u_\rho^{P_{m\rho}+1}$

  If $\hat{Q}^{\hat{a}}_m$ span *cokernel* of $\Pi_{m\rho}$, then $\widehat{T}_{\hat{a}} \equiv \prod_{m=1}^{u} \left[A_m A_0^{-1}\right]^{\hat{Q}^{\hat{a}}_m}$ are invariant.

The (2,2) GLSM moduli space

- T_a : toric Kähler deformations $H^{1,1}_{\text{toric}}(M) \subseteq H^{1,1}(M)$

  Toric divisors need not intersect $M \implies$ some T_a are redundant.
- A_m : polynomial c-x structure defs $H^{d-2,1}_{\text{poly}}(M) \subseteq H^{d-2,1}(M)$

  Redefinitions $Z_\rho \mapsto U_\rho(Z)$ act on $A_m \implies$ some A_m are redundant.

  Example: $Z_\rho \mapsto u_\rho Z_\rho \implies A_m \mapsto A_m \times \prod_\rho u_\rho^{P_{m\rho}+1}$

  If $\hat{Q}^{\hat{a}}_m$ span *cokernel* of $\Pi_{m\rho}$, then $\widehat{T}_{\hat{a}} \equiv \prod_{m=1}^{u} \left[A_m A_0^{-1}\right]^{\hat{Q}^{\hat{a}}_m}$ are invariant.

  Additional redefinitions lead to further redundancy in the $\widehat{T}_{\hat{a}}$.

The (2,2) GLSM moduli space

- T_a : toric Kähler deformations $\qquad H^{1,1}_{\text{toric}}(M) \subseteq H^{1,1}(M)$

  Toric divisors need not intersect $M \implies$ some T_a are redundant.

- A_m : polynomial c-x structure defs $\qquad H^{d-2,1}_{\text{poly}}(M) \subseteq H^{d-2,1}(M)$

  Redefinitions $Z_\rho \mapsto U_\rho(Z)$ act on $A_m \implies$ some A_m are redundant.

  Example: $\quad Z_\rho \mapsto u_\rho Z_\rho \implies A_m \mapsto A_m \times \prod_\rho u_\rho^{P_{m\rho}+1}$

  If $\hat{Q}^{\hat{a}}_m$ span *cokernel* of $\Pi_{m\rho}$, then $\widehat{T}_{\hat{a}} \equiv \prod_{m=1}^{u} \left[A_m A_0^{-1}\right]^{\hat{Q}^{\hat{a}}_m}$ are invariant.

  Additional redefinitions lead to further redundancy in the $\widehat{T}_{\hat{a}}$.

The Mirror Map for toric/polynomial deformations

$$\underbrace{\Pi, Q; T_a, \widehat{T}_{\hat{a}}}_{M\subset V} \quad \text{is mirror to} \quad \underbrace{\Pi^T, \hat{Q}; \widehat{T}_{\hat{a}}, T_a}_{M^\circ \subset V^\circ}$$

Redundant T_a, $\widehat{T}_{\hat{a}}$ from polytopes

No additional redundancy iff polytope is reflexively plain.

Redundant T_a, $\widehat{T}_{\hat{a}}$ from polytopes

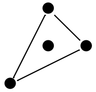
No additional redundancy iff polytope is reflexively plain.

- A polytope is plain iff it has no interior lattice points in any facet.

Redundant T_a, $\widehat{T}_{\hat{a}}$ from polytopes

No additional redundancy iff polytope is reflexively plain.

- A polytope is plain iff it has no interior lattice points in any facet.

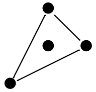


a plain polytope in $d = 2$

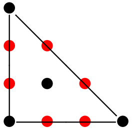
Redundant T_a, $\widehat{T}_{\hat{a}}$ from polytopes

No additional redundancy iff polytope is reflexively plain.

- A polytope is plain iff it has no interior lattice points in any facet.



a plain polytope in $d = 2$

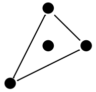


with a non-plain dual polytope

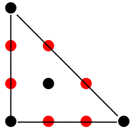
Redundant T_a, $\widehat{T}_{\hat{a}}$ from polytopes

No additional redundancy iff polytope is reflexively plain.

- A polytope is plain iff it has no interior lattice points in any facet.



a plain polytope in $d = 2$



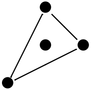
with a non-plain dual polytope

- A polytope is reflexively plain iff it and its dual are plain.

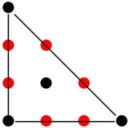
Redundant $T_a, \widehat{T}_{\hat{a}}$ from polytopes

No additional redundancy iff polytope is reflexively plain.

- A polytope is plain iff it has no interior lattice points in any facet.

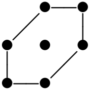


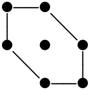
a plain polytope in $d = 2$



with a non-plain dual polytope

- A polytope is reflexively plain iff it and its dual are plain.
  - $d = 2$: there is a unique reflexively plain reflexive polytope:

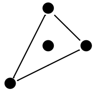




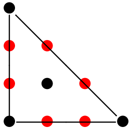
Redundant T_a, $\widehat{T}_{\hat{a}}$ from polytopes

No additional redundancy iff polytope is reflexively plain.

- A polytope is plain iff it has no interior lattice points in any facet.



a plain polytope in $d = 2$



with a non-plain dual polytope

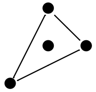
- A polytope is reflexively plain iff it and its dual are plain.
  - $d = 2$: there is a unique reflexively plain reflexive polytope:

  - $d = 4$: there are $6,677,743$ reflexively plain pairs and $5,518$ self-dual plain polytopes.

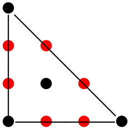
Redundant T_a, $\widehat{T}_{\hat{a}}$ from polytopes

No additional redundancy iff polytope is reflexively plain.

- A polytope is plain iff it has no interior lattice points in any facet.



a plain polytope in $d = 2$



with a non-plain dual polytope

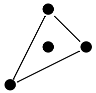
- A polytope is reflexively plain iff it and its dual are plain.

- Redundant T_a, $\widehat{T}_{\hat{a}}$ correspond to interior lattice points of facets;

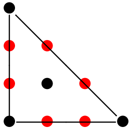
Redundant $T_a, \widehat{T}_{\hat{a}}$ from polytopes

No additional redundancy iff polytope is reflexively plain.

- A polytope is plain iff it has no interior lattice points in any facet.



a plain polytope in $d = 2$



with a non-plain dual polytope

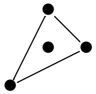
- A polytope is reflexively plain iff it and its dual are plain.

- Redundant $T_a, \widehat{T}_{\hat{a}}$ correspond to interior lattice points of facets;
- the redundancy is mirror symmetric;

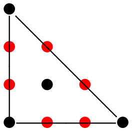
Redundant T_a, $\widehat{T}_{\hat{a}}$ from polytopes

No additional redundancy iff polytope is reflexively plain.

- A polytope is plain iff it has no interior lattice points in any facet.



a plain polytope in $d = 2$



with a non-plain dual polytope

- A polytope is reflexively plain iff it and its dual are plain.
- Redundant T_a, $\widehat{T}_{\hat{a}}$ correspond to interior lattice points of facets;
- the redundancy is mirror symmetric;
- (0,2) deformations take simple form in reflexively plain models.

(0,2) GLSM deformations

(0,2) GLSM deformations

- (2,2) vectors $\mapsto$ (0,2) vectors $+$ (0,2) bosonic chiral multiplets S_α

(0,2) GLSM deformations

- (2,2) vectors $\mapsto$ (0,2) vectors + (0,2) bosonic chiral multiplets S_α
- (2,2) chiral multiplets $\rightarrow$ (0,2) multiplets: $Z_\rho^{(2,2)} \mapsto (Z_\rho, \Gamma^\rho)$

(0,2) GLSM deformations

- (2,2) vectors $\mapsto$ (0,2) vectors + (0,2) bosonic chiral multiplets S_α
- (2,2) chiral multiplets $\rightarrow$ (0,2) multiplets: $Z_\rho^{(2,2)} \mapsto (Z_\rho, \Gamma^\rho)$
  - Z_ρ : (0,2) bosonic chiral multiplets
  - Γ^ρ : (0,2) Fermi multiplets $\qquad \overline{D}\Gamma^\rho = \sum_\alpha S_\alpha E^{\alpha\rho}(Z)$

(0,2) GLSM deformations

- (2,2) vectors $\mapsto$ (0,2) vectors + (0,2) bosonic chiral multiplets S_α
- (2,2) chiral multiplets $\to$ (0,2) multiplets: $Z_\rho^{(2,2)} \mapsto (Z_\rho, \Gamma^\rho)$
  - Z_ρ : (0,2) bosonic chiral multiplets
  - Γ^ρ : (0,2) Fermi multiplets $\overline{D}\Gamma^\rho = \sum_\alpha S_\alpha E^{\alpha\rho}(Z)$
- (2,2) superpotential $\mapsto$ (0,2) superpotential

$$W(Z) \mapsto \Gamma^0 F(Z) + \sum_\rho \Gamma^\rho Z_0 J_\rho(Z)$$

(0,2) GLSM deformations

- (2,2) vectors $\mapsto$ (0,2) vectors + (0,2) bosonic chiral multiplets S_α
- (2,2) chiral multiplets $\to$ (0,2) multiplets: $Z_\rho^{(2,2)} \mapsto (Z_\rho, \Gamma^\rho)$
  - Z_ρ : (0,2) bosonic chiral multiplets
  - Γ^ρ : (0,2) Fermi multiplets $\overline{D}\Gamma^\rho = \sum_\alpha S_\alpha E^{\alpha\rho}(Z)$
- (2,2) superpotential $\mapsto$ (0,2) superpotential

$$W(Z) \mapsto \Gamma^0 F(Z) + \sum_\rho \Gamma^\rho Z_0 J_\rho(Z)$$

- supersymmetry constraint:

$$Z_0 \sum_\rho E^{\alpha\rho}(Z) J_\rho(Z) + E^{\alpha 0} F(Z) = 0$$

(0,2) GLSM deformations

- (2,2) vectors $\mapsto$ (0,2) vectors + (0,2) bosonic chiral multiplets S_α
- (2,2) chiral multiplets $\to$ (0,2) multiplets: $Z_\rho^{(2,2)} \mapsto (Z_\rho, \Gamma^\rho)$
  - Z_ρ : (0,2) bosonic chiral multiplets
  - Γ^ρ : (0,2) Fermi multiplets $\quad \overline{D}\Gamma^\rho = \sum_\alpha S_\alpha E^{\alpha\rho}(Z)$
- (2,2) superpotential $\mapsto$ (0,2) superpotential

$$W(Z) \mapsto \Gamma^0 F(Z) + \sum_\rho \Gamma^\rho Z_0 J_\rho(Z)$$

- supersymmetry constraint:

$$Z_0 \sum_\rho E^{\alpha\rho}(Z) J_\rho(Z) + E^{\alpha 0} F(Z) = 0$$

- (2,2) locus: $\quad E^\rho = \sum_\alpha S_\alpha Q_\rho^\alpha Z_\rho, \qquad J_\rho = \dfrac{\partial F}{\partial Z_\rho}$

Counting (0,2) GLSM deformations [Kreuzer, McOrist, IVM + Plesser, 2010]

Combinatorics determines the number of parameters and redefinitions.

Counting (0,2) GLSM deformations [Kreuzer, McOrist, IVM + Plesser, 2010]

Combinatorics determines the number of parameters and redefinitions.

- Parameters: T_a, A_m and coefficients in $E^{\alpha\rho}, J_\rho$

Counting (0,2) GLSM deformations [Kreuzer, McOrist, IVM + Plesser, 2010]

Combinatorics determines the number of parameters and redefinitions.

- Parameters: T_a, A_m and coefficients in $E^{\alpha\rho}, J_\rho$

  $\#(E^{\alpha\rho}) = \#(\text{monomials of same charge as } Z_\rho)$

  $\#(J_\rho) \quad = \#(\text{monomials of same charge as } \partial F/\partial Z_\rho)$

Counting (0,2) GLSM deformations [Kreuzer, McOrist, IVM + Plesser, 2010]

Combinatorics determines the number of parameters and redefinitions.

- Parameters: T_a, A_m and coefficients in $E^{\alpha\rho}, J_\rho$
  $\#(E^{\alpha\rho}) = \#(\text{monomials of same charge as } Z_\rho)$
  $\#(J_\rho) \;\; = \#(\text{monomials of same charge as } \partial F/\partial Z_\rho)$
- SUSY constraint eliminates some of these

Counting (0,2) GLSM deformations [Kreuzer, McOrist, IVM + Plesser, 2010]

Combinatorics determines the number of parameters and redefinitions.

- Parameters: T_a, A_m and coefficients in $E^{\alpha\rho}, J_\rho$
  $\#(E^{\alpha\rho}) = \#(\text{monomials of same charge as } Z_\rho)$
  $\#(J_\rho) \quad = \#(\text{monomials of same charge as } \partial F/\partial Z_\rho)$
- SUSY constraint eliminates some of these
- As do redefinitions of Z_ρ, Γ^ρ and S_α

Counting (0,2) GLSM deformations [Kreuzer, McOrist, IVM + Plesser, 2010]

Combinatorics determines the number of parameters and redefinitions.

- Parameters: T_a, A_m and coefficients in $E^{\alpha\rho}, J_\rho$
  $\#(E^{\alpha\rho}) = \#(\text{monomials of same charge as } Z_\rho)$
  $\#(J_\rho) \; = \#(\text{monomials of same charge as } \partial F/\partial Z_\rho)$
- SUSY constraint eliminates some of these
- As do redefinitions of Z_ρ, Γ^ρ and S_α
- Result: $N(M)$—the number of GLSM deformations

Counting (0,2) GLSM deformations [Kreuzer, McOrist, IVM + Plesser, 2010]

Combinatorics determines the number of parameters and redefinitions.

- Parameters: T_a, A_m and coefficients in $E^{\alpha\rho}, J_\rho$
  $\#(E^{\alpha\rho}) = \#(\text{monomials of same charge as } Z_\rho)$
  $\#(J_\rho) \quad = \#(\text{monomials of same charge as } \partial F/\partial Z_\rho)$
- SUSY constraint eliminates some of these
- As do redefinitions of Z_ρ, Γ^ρ and S_α
- Result: $N(M)$—the number of GLSM deformations
- Check: $N(M)$ matches geometric computations in examples.

Counting (0,2) GLSM deformations [Kreuzer, McOrist, IVM + Plesser, 2010]

Combinatorics determines the number of parameters and redefinitions.

- Parameters: T_a, A_m and coefficients in $E^{\alpha\rho}, J_\rho$
  $\#(E^{\alpha\rho}) = \#(\text{monomials of same charge as } Z_\rho)$
  $\#(J_\rho) \quad = \#(\text{monomials of same charge as } \partial F/\partial Z_\rho)$
- SUSY constraint eliminates some of these
- As do redefinitions of Z_ρ, Γ^ρ and S_α
- Result: $N(M)$—the number of GLSM deformations
- Check: $N(M)$ matches geometric computations in examples.

A (0,2) GLSM mirror map for pair M, M° ?

Counting (0,2) GLSM deformations [Kreuzer, McOrist, IVM + Plesser, 2010]

Combinatorics determines the number of parameters and redefinitions.

- Parameters: T_a, A_m and coefficients in $E^{\alpha\rho}, J_\rho$
  $\#(E^{\alpha\rho}) = \#(\text{monomials of same charge as } Z_\rho)$
  $\#(J_\rho) \quad = \#(\text{monomials of same charge as } \partial F/\partial Z_\rho)$
- SUSY constraint eliminates some of these
- As do redefinitions of Z_ρ, Γ^ρ and S_α
- Result: $N(M)$—the number of GLSM deformations
- Check: $N(M)$ matches geometric computations in examples.

A (0,2) GLSM mirror map for pair M, M° ?

$$N(M) \stackrel{?}{=} N(M^\circ)$$

Counting (0,2) GLSM deformations [Kreuzer, McOrist, IVM + Plesser, 2010]

Combinatorics determines the number of parameters and redefinitions.

- Parameters: T_a, A_m and coefficients in $E^{\alpha\rho}, J_\rho$
  $\#(E^{\alpha\rho}) = \#(\text{monomials of same charge as } Z_\rho)$
  $\#(J_\rho) \;\; = \#(\text{monomials of same charge as } \partial F/\partial Z_\rho)$
- SUSY constraint eliminates some of these
- As do redefinitions of Z_ρ, Γ^ρ and S_α
- Result: $N(M)$—the number of GLSM deformations
- Check: $N(M)$ matches geometric computations in examples.

A (0,2) GLSM mirror map for pair M, M° ?

$$N(M) \overset{?}{=} N(M^\circ)$$

In general, no. If model is reflexively plain, yes!

Deformations of reflexively plain GLSMs

Deformations of reflexively plain GLSMs

Write $Z_\rho J_\rho = \sum_{m=0}^{u} L_{m\rho} \prod_\lambda Z_\lambda^{P_{m\lambda}+1}, \quad L_{m\rho} = 0 \text{ if } P_{m\rho} = -1$

Deformations of reflexively plain GLSMs

Write $Z_\rho J_\rho = \sum_{m=0}^{u} L_{m\rho} \prod_\lambda Z_\lambda^{P_{m\lambda}+1}, \quad L_{m\rho} = 0 \text{ if } P_{m\rho} = -1$

Invariant parameters

Deformations of reflexively plain GLSMs

Write $Z_\rho J_\rho = \sum_{m=0}^{u} L_{m\rho} \prod_\lambda Z_\lambda^{P_{m\lambda}+1}, \quad L_{m\rho} = 0 \text{ if } P_{m\rho} = -1$

Invariant parameters

- $\widehat{K}_{\hat{a}} \equiv \prod_{m=1}^{u} \left[A_m A_0^{-1}\right]^{\widehat{Q}_m^{\hat{a}}}$ complex structure

Deformations of reflexively plain GLSMs

Write $Z_\rho J_\rho = \sum_{m=0}^{u} L_{m\rho} \prod_\lambda Z_\lambda^{P_{m\lambda}+1}, \quad L_{m\rho} = 0 \text{ if } P_{m\rho} = -1$

Invariant parameters

- $\widehat{K}_{\hat{a}} \equiv \prod_{m=1}^{u} \left[A_m A_0^{-1}\right]^{\widehat{Q}_m^{\hat{a}}}$ complex structure
- $K_a \equiv T_a \prod_{\rho=1}^{n} \left[L_{0\rho} A_0^{-1}\right]^{Q_\rho^a}$ Kähler

Deformations of reflexively plain GLSMs

Write $Z_\rho J_\rho = \sum_{m=0}^{u} L_{m\rho} \prod_\lambda Z_\lambda^{P_{m\lambda}+1}, \quad L_{m\rho} = 0 \text{ if } P_{m\rho} = -1$

Invariant parameters

- $\widehat{K}_{\hat{a}} \equiv \prod_{m=1}^{u} \left[A_m A_0^{-1}\right]^{\widehat{Q}_m^{\hat{a}}}$ complex structure
- $K_a \equiv T_a \prod_{\rho=1}^{n} \left[L_{0\rho} A_0^{-1}\right]^{Q_\rho^a}$ Kähler
- $B_{m\rho} \equiv \frac{A_0 L_{m\rho}}{A_m L_{0\rho}} - 1$ for $m \neq 0$ bundle

Deformations of reflexively plain GLSMs

Write $Z_\rho J_\rho = \sum_{m=0}^{u} L_{m\rho} \prod_\lambda Z_\lambda^{P_{m\lambda}+1}, \quad L_{m\rho} = 0 \text{ if } P_{m\rho} = -1$

Invariant parameters

- $\widehat{K}_{\hat{a}} \equiv \prod_{m=1}^{u} \left[A_m A_0^{-1}\right]^{\widehat{Q}_m^{\hat{a}}}$ complex structure
- $K_a \equiv T_a \prod_{\rho=1}^{n} \left[L_{0\rho} A_0^{-1}\right]^{Q_\rho^a}$ Kähler
- $B_{m\rho} \equiv \frac{A_0 L_{m\rho}}{A_m L_{0\rho}} - 1$ for $m \neq 0$ bundle

$B_{m\rho}$ is rank d and satisfies $B_{m\rho} = -1$ whenever $\Pi_{m\rho} = -1$.

Deformations of reflexively plain GLSMs

Write $Z_\rho J_\rho = \sum_{m=0}^{u} L_{m\rho} \prod_\lambda Z_\lambda^{P_{m\lambda}+1}, \quad L_{m\rho} = 0 \text{ if } P_{m\rho} = -1$

Invariant parameters

- $\widehat{K}_{\hat{a}} \equiv \prod_{m=1}^{u} \left[A_m A_0^{-1}\right]^{\widehat{Q}_m^{\hat{a}}}$ complex structure
- $K_a \equiv T_a \prod_{\rho=1}^{n} \left[L_{0\rho} A_0^{-1}\right]^{Q_\rho^a}$ Kähler
- $B_{m\rho} \equiv \frac{A_0 L_{m\rho}}{A_m L_{0\rho}} - 1$ for $m \neq 0$ bundle

$B_{m\rho}$ is rank d and satisfies $B_{m\rho} = -1$ whenever $\Pi_{m\rho} = -1$.

Remaining parameters fixed by SUSY constraint up to redefinitions

(0,2) Mirror symmetry for reflexively plain GLSMs

(0,2) Mirror symmetry for reflexively plain GLSMs

Algebraic coordinates

(0,2) Mirror symmetry for reflexively plain GLSMs

Algebraic coordinates

- K_a : $n - d$ "Kähler" parameters. (2,2) locus: $K_a = T_a$.

(0,2) Mirror symmetry for reflexively plain GLSMs

Algebraic coordinates

- K_a : $n - d$ "Kähler" parameters. (2,2) locus: $K_a = T_a$.
- $\widehat{K}_{\hat{a}}$: $u - d$ "complex structure" parameters. (2,2) locus: $\widehat{K}_{\hat{a}} = \widehat{T}_{\hat{a}}$.

(0,2) Mirror symmetry for reflexively plain GLSMs

Algebraic coordinates

- K_a : $n - d$ "Kähler" parameters. (2,2) locus: $K_a = T_a$.
- $\widehat{K}_{\hat{a}}$: $u - d$ "complex structure" parameters. (2,2) locus: $\widehat{K}_{\hat{a}} = \widehat{T}_{\hat{a}}$.
- $B_{m\rho}$: bundle parameters. (2,2) locus: $B_{m\rho} = \Pi_{m\rho}$.

(0,2) Mirror symmetry for reflexively plain GLSMs

Algebraic coordinates

- K_a : $n - d$ "Kähler" parameters. (2,2) locus: $K_a = T_a$.
- $\widehat{K}_{\hat{a}}$: $u - d$ "complex structure" parameters. (2,2) locus: $\widehat{K}_{\hat{a}} = \widehat{T}_{\hat{a}}$.
- $B_{m\rho}$: bundle parameters. (2,2) locus: $B_{m\rho} = \Pi_{m\rho}$.
- number of parameters matches $N(M)$.

(0,2) Mirror symmetry for reflexively plain GLSMs

Algebraic coordinates

- K_a : $n - d$ "Kähler" parameters. (2,2) locus: $K_a = T_a$.
- $\widehat{K}_{\hat{a}}$: $u - d$ "complex structure" parameters. (2,2) locus: $\widehat{K}_{\hat{a}} = \widehat{T}_{\hat{a}}$.
- $B_{m\rho}$: bundle parameters. (2,2) locus: $B_{m\rho} = \Pi_{m\rho}$.
- number of parameters matches $N(M)$.

The (0,2) mirror conjecture

$$\underbrace{\Pi, Q; K_a, \widehat{K}_{\hat{a}}, B}_{(M,\mathcal{E})} \quad \text{is mirror to} \quad \underbrace{\Pi^T, \hat{Q}; \widehat{K}_{\hat{a}}, K_a, B^T}_{(M^\circ,\mathcal{E}^\circ)}.$$

(0,2) Mirror symmetry for reflexively plain GLSMs

Algebraic coordinates

- K_a : $n - d$ "Kähler" parameters. (2,2) locus: $K_a = T_a$.
- $\widehat{K}_{\hat{a}}$: $u - d$ "complex structure" parameters. (2,2) locus: $\widehat{K}_{\hat{a}} = \widehat{T}_{\hat{a}}$.
- $B_{m\rho}$: bundle parameters. (2,2) locus: $B_{m\rho} = \Pi_{m\rho}$.
- number of parameters matches $N(M)$.

The (0,2) mirror conjecture

$$\underbrace{\Pi, Q; K_a, \widehat{K}_{\hat{a}}, B}_{(M,\mathcal{E})} \quad \text{is mirror to} \quad \underbrace{\Pi^T, \hat{Q}; \widehat{K}_{\hat{a}}, K_a, B^T}_{(M^\circ,\mathcal{E}^\circ)} .$$

A Test: A/2 and B/2 singular loci

- A/2-twisted correlators diverge when an S_α develops a flat direction;

(0,2) Mirror symmetry for reflexively plain GLSMs

Algebraic coordinates

- K_a : $n - d$ "Kähler" parameters. (2,2) locus: $K_a = T_a$.
- $\widehat{K}_{\hat{a}}$: $u - d$ "complex structure" parameters. (2,2) locus: $\widehat{K}_{\hat{a}} = \widehat{T}_{\hat{a}}$.
- $B_{m\rho}$: bundle parameters. (2,2) locus: $B_{m\rho} = \Pi_{m\rho}$.
- number of parameters matches $N(M)$.

The (0,2) mirror conjecture

$$\underbrace{\Pi, Q; K_a, \widehat{K}_{\hat{a}}, B}_{(M,\mathcal{E})} \quad \text{is mirror to} \quad \underbrace{\Pi^T, \hat{Q}; \widehat{K}_{\hat{a}}, K_a, B^T}_{(M^\circ,\mathcal{E}^\circ)}.$$

A Test: A/2 and B/2 singular loci

- A/2-twisted correlators diverge when an S_α develops a flat direction;
- B/2-twisted correlators diverge when Z_0 develops a flat direction;

(0,2) Mirror symmetry for reflexively plain GLSMs

Algebraic coordinates

- K_a : $n - d$ "Kähler" parameters. (2,2) locus: $K_a = T_a$.
- $\widehat{K}_{\hat{a}}$: $u - d$ "complex structure" parameters. (2,2) locus: $\widehat{K}_{\hat{a}} = \widehat{T}_{\hat{a}}$.
- $B_{m\rho}$: bundle parameters. (2,2) locus: $B_{m\rho} = \Pi_{m\rho}$.
- number of parameters matches $N(M)$.

The (0,2) mirror conjecture

$$\underbrace{\Pi, Q; K_a, \widehat{K}_{\hat{a}}, B}_{(M,\mathcal{E})} \quad \text{is mirror to} \quad \underbrace{\Pi^T, \hat{Q}; \widehat{K}_{\hat{a}}, K_a, B^T}_{(M^\circ,\mathcal{E}^\circ)} .$$

A Test: A/2 and B/2 singular loci

- A/2-twisted correlators diverge when an S_α develops a flat direction;
- B/2-twisted correlators diverge when Z_0 develops a flat direction;
- these singular loci are exchanged by the mirror map.

Summary and Outlook

Summary and Outlook

Results

Summary and Outlook

Results

- GLSM offers hands-on description of a subclass of bundle deformations.

Summary and Outlook

Results

- GLSM offers hands-on description of a subclass of bundle deformations.
- In the reflexively plain class of models, there is a simple mirror map.

Summary and Outlook

Results

- GLSM offers hands-on description of a subclass of bundle deformations.
- In the reflexively plain class of models, there is a simple mirror map.
- More generally, map yields isomorphism of subfamilies of (0,2) GLSMs with only some of the E-deformations turned on.

Summary and Outlook

Results

- GLSM offers hands-on description of a subclass of bundle deformations.
- In the reflexively plain class of models, there is a simple mirror map.
- More generally, map yields isomorphism of subfamilies of (0,2) GLSMs with only some of the E-deformations turned on.

Questions

- Are A/2 and B/2 correlators exchanged by the mirror map?
- How to incorporate additional E-deformations?
- What is the fate of the non-GLSM bundle deformations?
- What is the space-time physics of the singularities?
- Can the ideas be generalized to (0,2) models without (2,2) locus?