Linear Sigma Models and Heterotic Moduli Spaces

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based on work with M. Kreuzer, J. McOrist, and M.R. Plesser
Motivation

A future textbook problem

Given a perturbative heterotic string background with $d = 4$, $N = 1$ super-Poincaré invariance, determine

1. moduli space & massless spectrum;
2. Yukawa coupling dependence on moduli fields;
3. the singular locus of CFT.

Extra credit

Apply your results to

- issues in moduli stabilization;
- non-perturbative effects in heterotic string theory;
- quantum geometry.
At present, difficult even with “textbook” starting point:

- Calabi-Yau three-fold $\mathcal{M}$, a hypersurface in a toric variety $\mathcal{V}$;
- standard embedding, i.e. holomorphic bundle $\mathcal{E} = T_{\mathcal{M}}$. 

Difficulty: bundle deformations and quantum corrections

Some special cases:
- special points in moduli space admit exact (0,2) SCFT description;
- $\mathcal{E} = T_{\mathcal{M}} \Rightarrow (2,2)$ world-sheet SUSY & mirror symmetry;
- If (2,2) theory admits gauged linear sigma model description, can study the (0,2) GLSM subspace of deformations.

A guiding question:

how does mirror symmetry extend to (0,2) GLSM deformations?
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Results from the (0,2) GLSM moduli space

- (0,2) deformations that are not lifted by instantons: Silverstein + Witten 1995, Berglund et al 1995, Basu + Sethi 2003, Beasley + Witten 2003
- Singular loci in moduli space: McOrist + IVM 2008, IVM + Plesser 2010
  ▶ points where SCFT expected to be singular
  ▶ interpolate between (2,2) singular loci and large radius bundle singularities
- Conjecture for a (0,2) mirror map: IVM + Plesser 2010
  ▶ suggested by form of algebraic coordinates
  ▶ \((M, E) \leftrightarrow (M^\circ, E^\circ)\)
  ▶ a check: map exchanges singular loci
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4. conjecture for a (0,2) mirror map [IVM + Plesser 2010]
   - suggested by form of algebraic coordinates
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The GLSM: a $d = 2$ (2,2) SUSY gauge theory

- gauge group $G = U(1)^r \times$ finite abelian group
- charged chiral matter multiplets $Z_0, Z_\rho, \rho = 1, \ldots, n$
- charges $Q_0^a, Q_\rho^a$, with $Q_0^a = -\sum_\rho Q_\rho^a$
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holomorphic parameters:

- $F\theta+\theta$-angle terms in twisted superpotential $T_a \equiv e^{-2\pi r^a+i\theta^a}$
- coefficients $A_m$ in chiral superpotential $W(Z) = Z_0 F(Z)$,

$$F(Z) = \sum_{m=0}^u A_m \prod_\rho Z_\rho^{P_{m\rho}+1}, \quad P_{m\rho} = \begin{cases} 0 \text{ if } m=0, \\ \Pi_{m\rho} \text{ otherwise.} \end{cases}$$
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\( Q^a_\rho, \Pi_{m\rho} \) determined by combinatorics of a reflexive polytope:

- \( \Pi_{m\rho} \): rank \( d = n - r \) integral \( u \times n \) matrix with entries \( \geq -1 \);
- \( \{ Q^1_\rho, \ldots, Q^r_\rho \} \): integral basis for kernel of \( \Pi_{m\rho} \).
The (2,2) GLSM & geometry

Combinatorics of $\Pi_{m\rho}$, $Q^a_{\rho} \rightarrow$ geometry:

- $Z_\rho$ are projective coordinates for $d$-dimensional compact toric variety 
  $V = \{\mathbb{C}^n - \Delta\}/G_{\mathbb{C}}$;
- $M = \{F = 0\} \subset V$ is a Calabi-Yau hypersurface in $V$. 
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GLSM/geometry connection:

- can choose $T_a$ so that at low energy GLSM reduces to NLSM with target-space $M$.
- just one of the phases of the GLSM.
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The GLSM parameters

- $T_a$ : Kähler deformations of $V \rightarrow$ Kähler deformations of $M$
- $A_m$ : complex structure deformations of $M$
The (2,2) GLSM moduli space
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- $T_a$: toric Kähler deformations

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  Example: $Z_{\rho} \leftrightarrow u_{\rho}Z_{\rho} \implies A_m \leftrightarrow A_m \times \prod_{\rho} u_{\rho}^{P_{m\rho}+1}$
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If $\hat{Q}_m$ span cokernel of $\Pi_{m\rho}$, then $\hat{T}_{\hat{a}} \equiv \prod_{m=1}^{u} \left[A_m A_0^{-1}\right] \hat{Q}_m$ are invariant.
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The Mirror Map for toric/polynomial deformations

$\Pi, Q; T_a, \hat{T}_\hat{a}$ is mirror to $\Pi^T, \hat{Q}; \hat{T}_\hat{a}, T_a$

$I.V. Melnikov (AEI)$

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Redundant $T_a, \hat{T}_\hat{a}$ from polytopes

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A plain polytope in $d = 2$
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![Diagram of a plain polytope in $d=2$ with a non-plain dual polytope](image)
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- A polytope is reflexively plain iff it and its dual are plain.
  - $d = 2$ : there is a unique reflexively plain reflexive polytope:
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A plain polytope in \(d = 2\) with a non-plain dual polytope:

- A polytope is **reflexively plain** iff it and its dual are plain.
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  - \(d = 4\) : there are 6,677,743 reflexively plain pairs and 5,518 self-dual plain polytopes.
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- (0,2) deformations take simple form in reflexively plain models.
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  - $Z_\rho$ : (0,2) bosonic chiral multiplets
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- (2,2) locus:
  \[
  E^\rho = \sum_\alpha S_\alpha Q_\rho^\alpha Z_\rho, \quad J_\rho = \frac{\partial F}{\partial Z_\rho}
  \]
Counting (0,2) GLSM deformations [Kreuzer, McOrist, IVM + Plesser, 2010]

Combinatorics determines the number of parameters and redefinitions.
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- SUSY constraint eliminates some of these

Check: $N(M)$ matches geometric computations in examples.

A (0,2) GLSM mirror map for pair $M, M^\circ$?

$N(M) = N(M^\circ)$ in general. If model is reflexively plain, yes!
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- Result: $N(M)$—the number of GLSM deformations

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In general, no. If model is reflexively plain, yes!
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In general, no. If model is reflexively plain, yes!
Deformations of reflexively plain GLSMs
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Write $Z_\rho J_\rho = \sum_{m=0}^{u} L_{m\rho} \prod_{\lambda} Z_\lambda^{P_{m\lambda}+1}$, $L_{m\rho} = 0$ if $P_{m\rho} = -1$.
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- \( \hat{K}_{\hat{a}} \equiv \prod_{m=1}^{u} \left[ A_m A_0^{-1} \right] \hat{Q}_m \) complex structure
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- $K_{a} \equiv T_{a} \prod_{\rho=1}^{n} \left[ L_{0\rho}A_{0}^{-1} \right] Q_{\rho}^{a}$ Kähler
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- $B_{m\rho} \equiv \frac{A_0L_{m\rho}}{A_mL_{0\rho}} - 1$ for $m \neq 0$ bundle
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\( B_{m\rho} \) is rank \( d \) and satisfies \( B_{m\rho} = -1 \) whenever \( \Pi_{m\rho} = -1 \).
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Remaining parameters fixed by SUSY constraint up to redefinitions
Mirror symmetry for reflexively plain GLSMs

Algebraic coordinates

$K_a$: $n - d$ “Kähler” parameters. (2,2) locus: $K_a = T_a$.

$\hat{K}_{\hat{a}}$: $u - d$ “complex structure” parameters. (2,2) locus: $\hat{K}_{\hat{a}} = \hat{T}_{\hat{a}}$.

$B_{m\rho}$: bundle parameters. (2,2) locus: $B_{m\rho} = \Pi_{m\rho}$.

number of parameters matches $N(M)$.

The (0,2) mirror conjecture

$\Pi, Q; K_a, \hat{K}_{\hat{a}}, B_T \leftrightarrow \Pi^T, \hat{Q}; \hat{K}_{\hat{a}}, K_a, B_T$. (M, E) is mirror to $(M^T, E^T)$.

A Test: A/2 and B/2 singular loci

A/2-twisted correlators diverge when an $S_{\alpha}$ develops a flat direction; B/2-twisted correlators diverge when $Z_0$ develops a flat direction; these singular loci are exchanged by the mirror map.
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\[ B_m^\rho : \text{bundle parameters.} \] (2,2) locus: \[ B_m^\rho = \prod m^\rho. \]

The (0,2) mirror conjecture \[ \Pi, \bar{Q}; K^a, \hat{K}^\hat{a}; B_T \] is mirror to \[ \Pi, \bar{Q}; \hat{K}^\hat{a}, K^a; B_T \].

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$$\Pi^T, \hat{Q}; \hat{K}_\hat{a}, K_a, B^T \quad \text{(M^o, E^o)}$$

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Summary and Outlook

GLSM offers hands-on description of a subclass of bundle deformations. In the reflexively plain class of models, there is a simple mirror map. More generally, map yields isomorphism of subfamilies of (0,2) GLSMs with only some of the E-deformations turned on.

Questions

- Are $A/2$ and $B/2$ correlators exchanged by the mirror map?
- How to incorporate additional E-deformations?
- What is the fate of the non-GLSM bundle deformations?
- What is the space-time physics of the singularities?
- Can the ideas be generalized to (0,2) models without (2,2) locus?
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